Control Barrier Function Based Speed Control for Fixed-Line Transit Headway Management

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***Abstract*— We present a control barrier function (CBF) approach for headway management in fixed-line transit systems through vehicular speed control. We frame headway management as a formal property in terms of vehicle time-differences. Using a dynamical model of vehicle speed control, we develop a combined CBF quadratic-programming (QP) supervisory approach. The CBF-QP supervises control speed inputs for forward-invariance of minimum and maximum time-differences between vehicles. While the need for vehicles to stop creates modeling error that can lead to property violations, the CBFs can recover back to satisfying the time-gap properties after violations. We present numerical experiments that compare the CBF-QP approach to other controllers. We find that the CBF-QP controller supervises underlying control algorithms to achieve significantly improved headway regularity. We additionally compare the CBF-QP with an existing linear-quadratic control approach and find that the CBF-QP performs similarly or better.**

I. Introduction

In fixed-line transit systems, vehicle bunching and gapping is known to lead to long passenger wait times, overcrowded buses, and lost time. Bunching is the phenomenon of transit vehicles drawing close to each other during operation. Gapping occurs when bunching subsequently creates large distances between vehicles. This leads to irregular differences in arrival times at stops.

Bunching and gapping can arise because of stochasticity in passenger demand at stops and variability in vehicular travel times. Schedule based transit operation that does not respond to external disturbances is known to suffer from bunching. Techniques for mitigating bunching are known as headway management [1].

Headway management techniques often focus on controlling the timing of vehicle operations to mitigate bunching. In holding approaches, vehicles remain at stops longer than needed for passenger pickup and alighting in order to correct emergent bunching [2]–[4]. In dispatching, vehicles can be deployed from a depot at variable times in order to respond to bunching [5].

A recent approach for headway management is through vehicular speed control. By continuously controlling the speed at which vehicles drive, bunching can be mitigated. Control theoretic approaches such as proportional control [6], linear-quadratic (LQ) control [6], and hybrid model predictive control [7] have shown promise.

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This work builds on such control theoretic approaches to speed based headway management. We develop a formal methods approach that uses control barrier functions (CBFs). CBFs are an established approach for formal safety based control [8], [9], and have found use in control of automated vehicles [10]–[13].CBFs control for forward invariance of formal properties by supervising underlying control algorithms. This is often solved numerically via quadratic-programming (QP) [8], [9].

In this work, we frame headway management in terms of sets of formal properties. We define a minimum and maximum time-gap difference range that we seek to keep vehicles within. This is done using CBFs. The time-gap CBFs can additionally supervise other underlying control algorithms, while supervising for time-gap property adherence. This approach allows for a more flexible problem statement in which headway management can be combined in a supervisory manner with other control approaches.

Contributions: The contributions of this work are threefold. First, we frame headway management as a formal control property based on the notion of forward-invariance. We do this by defining minimum and maximum desired inter-vehicle time-differences. While prior work has explored headway control as an optimization objective or a stability goal, this work is the first to approach it from a formal invariance perspective. Second, we show how control barrier functions (CBFs) can be used to create a supervisory control framework that manages for headway. Finally, we demonstrate the ability of the proposed CBF based approach to effectively perform headway management in a series of numerical experiments. We compare the proposed CBF approach to the prior LQ approach [14] and a "do-nothing" baseline. We find that the CBF performs similarly or better headway regularity performance across all experiments.

Organization: The organization of this work is as follows. In Section II, we provide background on fixed-line transit and the mathematics of CBFs. In Section III, we present the system model and formulate the headway-management problem. We introduce the control approach based on CBFs in Section IV, and present numerical experiments in Section V. We conclude the work and discuss future directions in Section VI.

II. Background

In this section, we first present a mathematical description of fixed-line transit systems. Second, we describe control barrier functions (CBFs).

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A. Fixed-Line Transit

In fixed-line transit systems, individual vehicles travel along fixed routes, stopping to collect passengers at discrete stops. Let $\mathbf{x}_i(t) = [p_i(t), v_i(t)]^T$ be the state of vehicle i at time t, where p is position along the transit line and v is velocity. Vehicles are indexed by i, which refers to the order of vehicles along the network, where vehicle $i-1$ precedes (leads) vehicle i(e.g., $p_i(t) < p_{i-1}(t)$). The state of the entire system can be described as $\mathbf{x} = [\mathbf{x}_n^T, \mathbf{x}_{n-1}^T, ...\mathbf{x}_1^T]^T$, where $\mathbf{x}_n$ is the state of the last vehicle (closest to start) in the network position-wise, and $\mathbf{x}_1$ the state of the vehicle at the front.

The instantaneous headway between a following and leading vehicle can be written as

$$T(x_i, x_{i-1}) = \frac{p_{i-1} - p_i}{v_i} \tag{1}$$

where $p_{i-1} - p_i$ is the inter-vehicle spacing difference. $T(x_i, x_{i-1})$ is the instantaneous time difference for the following vehicle to cover the current spacing difference to the leading vehicle at speed v. Controlling this time difference to remain in a given range is how headway management is done in this context.

B. Control Barrier Functions

Control barrier functions (CBFs) are an approach for active safety filtering to provide formal guarantees of forward-invariance with respect to a safety property. CBFS can be applied to control-affine functions (among other functions) of the following form:

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})\mathbf{u} \tag{2}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the system state, $\mathbf{u} \in \mathbb{R}^m$ are control inputs, and $f : \mathbb{R}^n \mapsto \mathbb{R}^n$, $g : \mathbb{R}^{m,n} \mapsto \mathbb{R}^n$ describe system dynamics.

CBFs can be applied to formal properties of the following form:

$$h(\mathbf{x}) \geq 0 \tag{3}$$

where $h : \mathbb{R}^n \mapsto \mathbb{R}^1$ is a continuous function mapping the system state to a scalar quantity. The property is satisfied when $h(\mathbf{x}) \geq 0$, meaning that each state $\mathbf{x}$ either satisfies or does not satisfy the property.

Forward-invariance of the property (3) can be stated as:

$$h(\mathbf{x}(t_0)) \geq 0 \implies h(\mathbf{x}(t)) \geq 0 \quad \forall t \geq 0 \tag{4}$$

meaning that if the property is satisfied at time t_0, then it will also be satisfied at all subsequent following times.

The function h is said to be a control-barrier function of Equation (2) if there exists a function $\alpha \in \text{class} - \mathscr{K}^e$ such that for all $\mathbf{x} \in \{\mathbf{x} : h(\mathbf{x}) \geq 0\}$:

$$\sup_{\mathbf{u}} : \dot{h}(\mathbf{x}, \mathbf{u}) + \alpha(h(\mathbf{x})) \geq 0 \tag{5}$$

where $\dot{h}$ is the time-derivative of h, which may be directly controllable via the control input $\mathbf{u}$. $\dot{h}$ is calculated using the Lie-derivative of h along the dynamics of Equation (2). For a more rigorous treatment of when calculating Lie-derivatives is possible, we direct the reader to [15], [16]. Any locally Lipschitz controller k satisfying (5) will cause $\mathbf{x}$ to be forward-invariant in $\{\mathbf{x} : h(\mathbf{x}) \geq 0\}$. This is often done by wrapping the constraint in (5) into a QP as follows:

$$\begin{aligned} \min_{u} \quad & \|u - u_{\text{nom}}(\mathbf{x})\| \\ \text{s.t.} \quad & \dot{h}(\mathbf{x}, \mathbf{u}) + \alpha(h(\mathbf{x})) \geq 0 \end{aligned} \tag{6}$$

where u_{nom} is a nominal control law that is supervised by the CBF to create property forward-invariance. Control chosen from a QP based control algorithm as in (6) will satisfy the condition in (5).

C. Recovery

In the event that $\mathbf{x}$ violates safety, it is desirable that the system recovers $\mathbf{x}$ back to safety. Recovery can be stated as:

$$h(\mathbf{x}) < 0 \implies \dot{h}(\mathbf{x}) > 0 \tag{7}$$

meaning that h is strictly increasing and thus stabilizing back into safety. Any controller k satisfying (5) for all $\mathbf{x} \in \{h(\mathbf{x}) < 0\}$ will satisfy recovery.

III. Model and Problem Statement

Here, we introduce the model used in this work and the ensuing problem statement. We first define how we model fixed-line transit systems and speed control. We then define minimum and maximum time-gap properties on the system state, which we use for for headway management. Finally, we describe the formal headway control problem in terms of forward-invariance of min/max time-gaps for all vehicles.

Remark: The model described in this section is interpretable in two ways. A first interpretation of this model is that of representing a human driver that manages the vehicle and accepts speed commands over time, which they try to follow in addition to managing stopping. The second interpretation of this model is that of an automated vehicle that manages speed control and stopping, and that accepts speed commands from the centralized control algorithm. In this way, the model—and the ensuing control approach—are generalizable to potentially both human operated vehicles and automated vehicles.

A. System Model

This work models vehicular dynamics along fixed-line networks in three modes: `DRIVING`, `STOPPING`, and `AT_STOP`. Each mode has corresponding dynamics and can transition to only one other mode.

In the `DRIVING` mode, vehicle i drives between stops and can receive speed control inputs. The dynamics are described via:

$$\dot{\mathbf{x}}_i = \begin{pmatrix} \dot{p}_i \\ \dot{v}_i \end{pmatrix} = \begin{pmatrix} v_i \\ \alpha_i(u_i - v_i) \end{pmatrix}, \tag{8}$$

where i indexes a given vehicle's position (p_i), speed (v_i), and commanded speed (u_i). The vehicle's responsiveness to a commanded speed is modeled via α_i, which could represent dynamics of a driver's response or the gain factor in a proportional control law. This model is very similar to the class of optimal-velocity models used in car-following models [17].

Additionally in DRIVING mode, upper and lower bounds of v_{max} and 0 are imposed on the vehicle speed, meaning that the vehicle cannot go arbitrarily fast or backward. The acceleration and deceleration is also bounded in the range $\dot{v}_i \in [\dot{v}_{min}, \dot{v}_{max}]\frac{m}{s^2}$, to prevent nonphysical/unrealistic accelerations/braking. In the numerical experiments in Section V, we use the range $[-3,3]\frac{m}{s^2}$.

In AT_STOP mode, the vehicle is stationary, i.e., both the vehicle's velocity and its acceleration are zero ($v_i = 0$ and $\dot{v}_i = 0$). The vehicle remains in AT_STOP for the duration of the boarding and alighting period, during which passengers enter and exit the vehicle. At the end of the period, the vehicle transitions to the DRIVING mode.

In STOPPING mode, the vehicle executes braking at a constant rate of a_{brake} to come to a stop. The STOPPING mode is entered from the DRIVING mode when $\Delta p_{stop,i}(t) \leq \frac{v_i^2(t)}{2a_{brake}}$, where $\Delta p_{stop,i}(t)$ is the positional distance to the next stop for vehicle i and $\frac{v_i^2(t)}{2a_{brake}}$ is the stopping distance under constant braking at rate a_{brake}. STOPPING mode transitions to AT_STOP when $v_i = 0$.

B. Problem Statement

We frame headway management here as a set of formal properties on the system state. Let the maximum desired headway time between vehicle i and vehicle $i-1$ be described via:

$$h_{max,i}(\mathbf{x}) = T_{max} \cdot v_i - (p_{i-1} - p_i) \tag{9}$$

where h_{max} is a distance in $[m]$. Let the minimum desired vehicle headway time-difference be described by:

$$h_{min,i}(\mathbf{x}) = (p_{i-1} - p_i) - T_{min} \cdot v_i \tag{10}$$

where T_{min} and T_{max} are minimum and maximum time differences. When $h_{min,i}(\mathbf{x}) \geq 0$, the minimum time-difference is satisfied for vehicle i; and when $h_{max,i}(\mathbf{x}) \geq 0$, the maximum time-difference is satisfied.

The goal of the control framework proposed in this work is to keep the state of each vehicle forward-invariant in both property (9) and property (10), which represent the prevention of both gapping and bunching. We state the problem formally as designing a control law $k(\mathbf{x})$ such that

$$\mathbf{u} = k(\mathbf{x}) \implies (h_{min,i}(\mathbf{x}(t)) \geq 0) \wedge (h_{max,i}(\mathbf{x}(t)) \geq 0) \quad \forall t, i \tag{11}$$

meaning that the control chosen by k will lead to forward-invariance in the two time-gap properties for all vehicles. We create a solution approach using control barrier functions.

IV. Control Approach

In this section, we present the design of the CBF-based headway control approach.

The time-derivatives of Equations (9) and (10) are:

$$\begin{aligned} \dot{h}_{max}(\mathbf{x}_i) &= T_{max}(\dot{v}) - \Delta v_i \\ &= T_{max}(\alpha_i(u_i - v_i)) - \Delta v_i \end{aligned}$$

and

$$\begin{aligned} \dot{h}_{min}(\mathbf{x}_i) &= \Delta v_i - T_{min}(\dot{v}) \\ &= \Delta v_i - T_{min}(\alpha_i(u_i - v_i)) \end{aligned}$$

This work uses positive scalar constants k as choices of class-$\mathscr{K}^e$ functions to form CBF filters for both headway properties and vehicles. For ease of notation, we only write k in the following description, but other choices—which could vary between different vehicles and properties—could be just as theoretically valid.

Using the condition for CBF forward-invariance defined in Equation (5), we write:

$$k \cdot h_{min,i}(\mathbf{x}) + \dot{h}_{min,i}(\mathbf{x}, \mathbf{u}) \geq 0$$

This is equivalent to:

$$k \cdot h_{min,i}(\mathbf{x}) + \Delta v_i - T_{min}(\alpha_i(u_i - v_i)) \geq 0$$

which is equivalent to:

$$u_i \leq v_i + \frac{1}{\alpha_i T_{min}}(k \cdot h_{min,i}(\mathbf{x}) + \Delta v_i) \tag{12}$$

which yields a constraint directly on u_i as a function of $\mathbf{x}$. For notational ease let the $u_{min,i}(\mathbf{x}_i)$ be the right-hand side of inequality (12).

The same can be done for $h_{max,i}$:

$$u_i \geq v_i + \frac{1}{\alpha_i T_{max}}(\Delta v_i - k \cdot h_{max,i}(\mathbf{x})) \tag{13}$$

where again, let $u_{max,i}(\mathbf{x}_i)$ refer to the right hand side of the inequality. $u_{max,i}(\mathbf{x}_i)$ and $u_{min,i}(\mathbf{x}_i)$ are upper and lower bounds on control speed defined at each vehicular state.

These can be combined into a unified quadratic program as:

$$\begin{aligned} \min_{u_i} \quad & \|u_i - u_{\text{nom}}(\mathbf{x})\|_2 \\ \text{s.t.} \quad & \begin{bmatrix} u_i \\ -u_i \end{bmatrix} \leq \begin{bmatrix} u_{min,i}(\mathbf{x}_i) \\ -u_{max,i}(\mathbf{x}_i) \end{bmatrix} \end{aligned} \tag{14}$$

The control approach in (14) is a flexible approach for supervising an underlying nominal control algorithm for headway management through a CBF-QP. The control from u_{nom} may be controllers that are not explicitly designed for headway management, and focus on other operational criteria. For instance, in section V we present an example of a CBF managing a simple speed maximizing controller. Nominal controllers that focus on operating vehicles to minimize fuel consumption or cost are also possible, or black-box approaches that lack rigorous guarantees of performance. The framework presented in this paper allows for supervising nominal controllers for headway management.

V. Numerical Examples

Here, we present numerical experiments quantifying the impact of the proposed CBF control approach.

A. Experimental Details

A synthetic network with evenly spaced stops is used for simulations. The network is $15[km]$ with stops every $500[m]$. Passengers arrive at stops randomly according to a specified demand level. Passengers have a boarding time of $3.5[s]$ and alighting time of $2[s]$. Origin-destination pairs are generated uniformly at random along the network.

In each simulation, vehicles are deployed from the beginning of the network every $300[s]$. Control algorithms are compared based on their ability to regulate headways at $300[s]$.

Controllers: Three controllers are compared across experiments. A maximum-velocity controller is used both as a baseline for a lack of headway management and as the nominal controller supervised by the CBF framework. This controller is of the form:

$$k_{nom,i}(\mathbf{x}) = v_{max} \tag{15}$$

where v_{max} is the maximum speed in the network. This controller is comparable to uncontrolled scenarios in which vehicles progress as fast as possible without information about other vehicles.

We create a CBF supervised controller by using the filter defined in equation (14) to supervised the the controller defined in (15). Parameter values of $k = 0.1$, $T_{min,i} = 240[s]$, $T_{max,i} = 360[s]$ are used. The value of k is chosen to be similar to other gain parameter choices in the CBF literature, but any positive choice of k is theoretically usable.

The third controller compared against is the linear-quadratic controller from [14], and is of the form:

$$\dot{v}_{des,i}(\mathbf{x}) = l_1(v_{i-1} - v_i) + l_2(p_{i-1} - p_i - T_{des}v_i) \tag{16}$$

where l_1 and l_2 are gain parameters and T_{des} a desired headway difference. In that work a direct control of acceleration was assumed. We adapt that control law to our model via:

$$u_i = v_i + \frac{1}{\alpha_i}\dot{v}_{des,i}(\mathbf{x})$$

which translates acceleration commands in to velocity commands. We use the same gain parameters provided in [14] in this work. A value of $T_{des} = 300[s]$ is used to match the desired headway on the network.

B. Numerical Results

Two sets of experiments are performed. In the first experimental set, the maximum velocity controller defined in equation (15) is compared against the CBF supervised controller.

In Figure 1, space-time trajectories of vehicles in each network are shown. In top plot, the max-velocity scenario vehicles experience bunching/gapping. The CBF controlled system is shown in the bottom plot where vehicles are more evenly spaced.

In Figure 2, the distribution of departure differences at all stops is shown. Departure difference bounds how long a passenger could have waited and is a good measure of arrival time regularity.

In Figure 3, distributions of values of $h_{min,i}$ and $h_{max,i}$ are shown. The corresponding speed commands from the minimum and maximum time-gap CBFs are shown in Figure 4.

In some cases, $h_{min,i} < 0$ and $h_{max,i} < 0$, meaning the properties are not satisfied. This is further explained in Figure 5. A vehicle's speed profile (v_i) over time is shown in addition to the upper and lower speeds specified by the

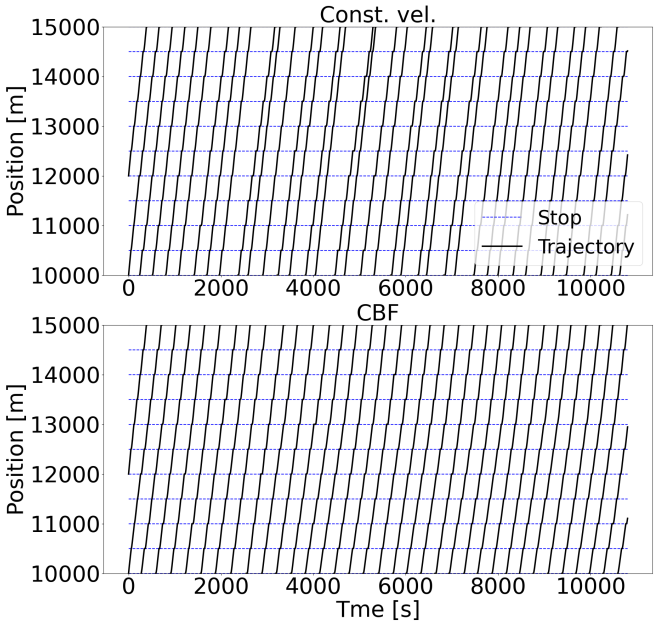

Fig. 1: Space-time trajectories for vehicles are shown (black) along the network with stops (blue). Top: trajectories of vehicles controlled by the max-velocity controller are shown. Bottom: trajectories of vehicles controlled by the CBF are shown.

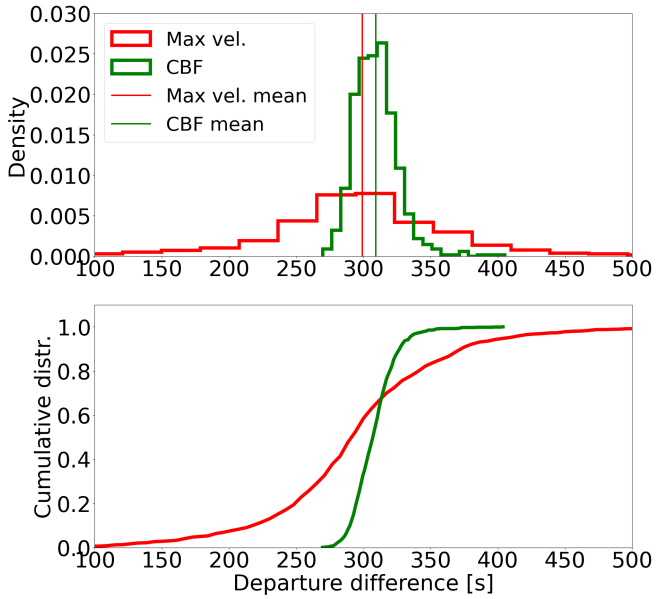

Fig. 2: Distributions of departure differences for all stops are shown. Top: departure differences in the CBF controlled case are shown in green, and the max velocity case in red. Bottom: The cumulative distribution of departure differences are shown.

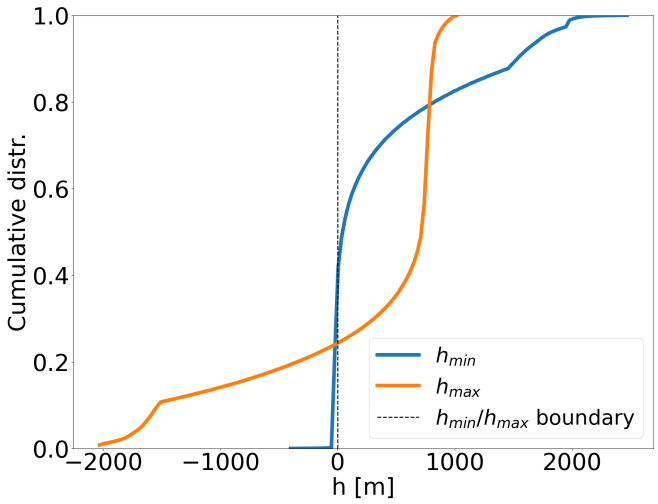

Fig. 3: The cumulative distributions of h_{min} and h_{max} are shown for experiment 1.

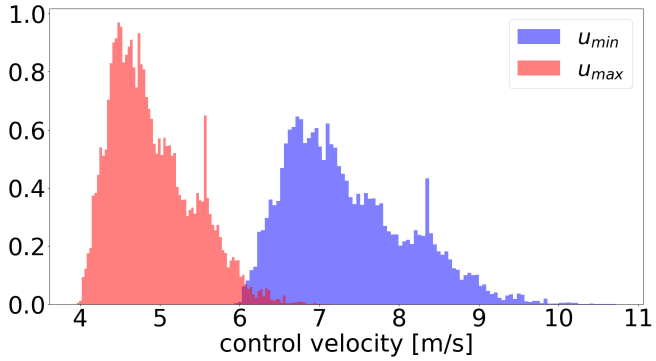

Fig. 4: The distributions of minimum and maximum allowed speeds by the CBF are shown for experiment 1.

CBF ($u_{max,i}, u_{min,i}$). During stops, the speed goes below the minimum speed allowed, violating the property. However, after the stop, the speed recovers back to within the admissible range, meaning that for much of the trajectory both properties are met.

The second set of experiments compare the CBF controller against the LQ controller defined in equation (16). The same synthetic network is used as in the first experiment. The demand level of passengers arriving at stops is varied across four levels: $(6, 30, 60, 120)$ passengers per hour arriving at every stop along the network. The results of this comparative analysis are shown in Figure 6.

In each case, the CBF performs as well or better than the LQ controller in terms of departure difference regularity. Distributions of departure differences are shown for each demand scenario. The mean and 95^{th} percentiles are shown as well. The CBF has a lower mean and 95^{th} percentile than the LQ controller in every example. As demand increases on the network the distribution of departure differences widens and shifts towards longer differences.

From these results, we conclude the following. The proposed CBF approach is able to supervise an otherwise poor-

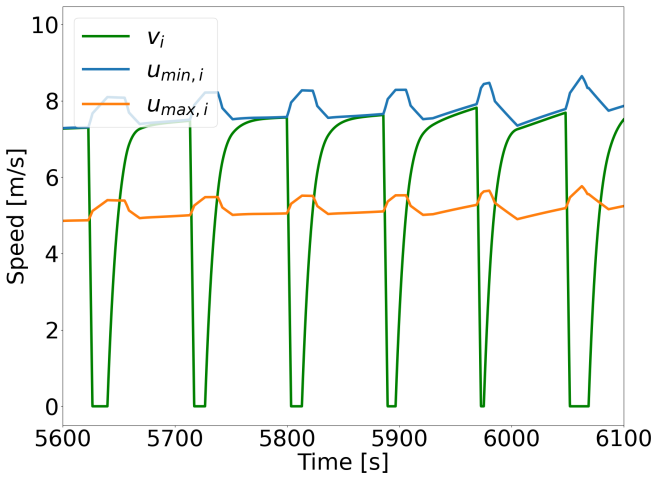

Fig. 5: Timeseries of one of the vehicle's speeds (v_i and upper and lower speeds specified by the CBF ($u_{max,i}, u_{min,i}$).

performing algorithm to enhance headway regularity. The need for vehicle's to stop, halting control, creates a challenge to forward-invariance, and at times the minimum and maximum time headway properties can be violated. Despite this, the approach is able to create stronger headway regularity by driving vehicles back towards property satisfaction. Finally, the CBF approach performs similarly or better than the LQ approach proposed in [14].

VI. Conclusions and Discussion

Here, we conclude the work and discuss future directions.

A. Conclusions

This work presents a control barrier function (CBF) based approach for managing headways in fixed-line transit systems using vehicular speed control. We frame headway regularity as a set of formal properties on the state of each vehicle. Using a dynamical model of vehicle speed control and a combined CBF quadratic program (QP) framework, we develop a control algorithm that supervises the system for maintaining headway differences between vehicles that satisfy the headway formal properties. We run a series of numerical experiments that compare the CBF approach to other control approaches. We show that the CBF is able to correct a maximum-velocity controller that does exhibit bunching to provide better headway regularity. We compare the CBF approach to an existing linear-quadratic approach and find that the CBF performs similarly to or better than the LQ approach in terms of headway regularity.

B. Future Directions

Several directions of future research are needed. Of particular interest is whether other control approaches exist (CBF based or other) that can more strictly achieve formal adherence to the min/max time-difference properties. Additionally, more research should be done on whether different choices of formal properties would lead to increased stop arrival/departure regularity. Finally, more realistic testing scenarios should be designed to quantify potential impact.

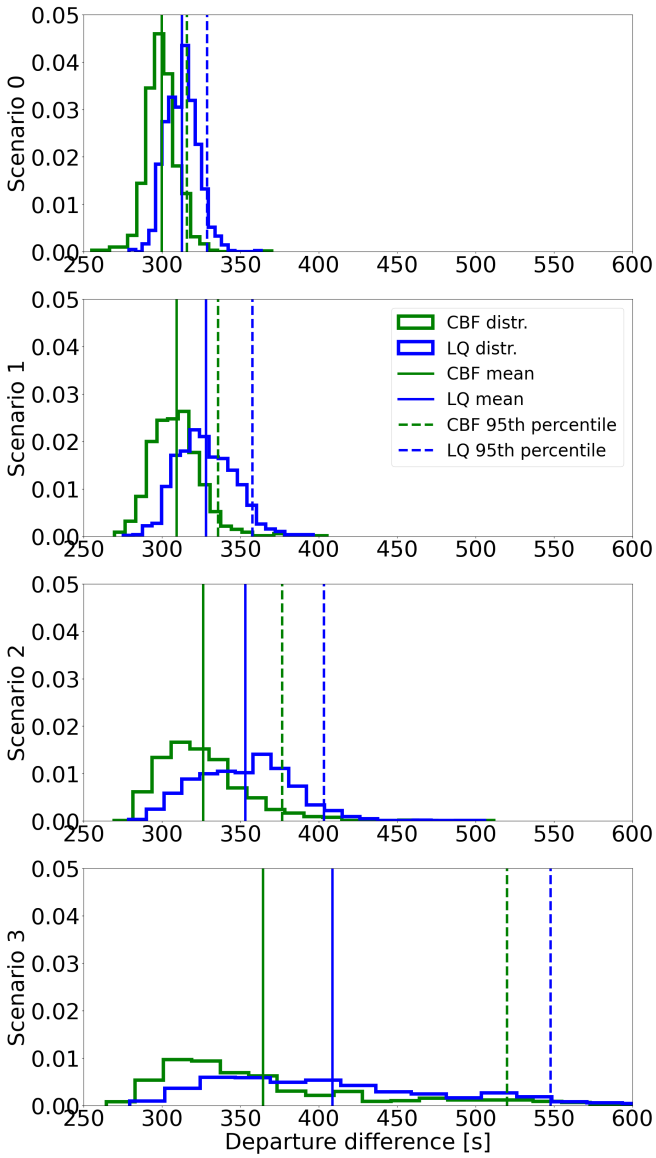

Fig. 6: Departure difference distributions are shown for all demand scenarios under CBF (blue) and LQ control (green). In each scenario the mean of each distribution is shown as well as the 95^{th} percentile.

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